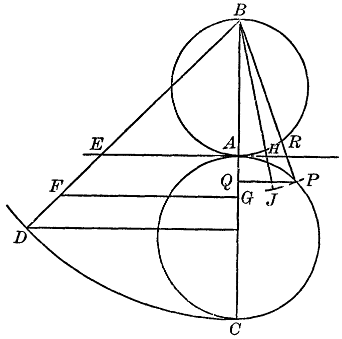
2734 [1918, 444]. Proposed by E. L. REES, University of Kentucky.

Given two circles tangent to each other externally. From the extremity of a diameter through the point of tangency draw a secant such that the segment between the circles shall be equal to a given segment.



SOLUTION BY THE PROPOSER.

Let PB be the required position of the secant, assuming the problem having been solved. Draw PQ perpendicular to AC and let $BP = y$, $AQ = x$, $AB = d$, $AC = d'$, and $RP = a$, the given segment. Then we have, $y^2 = (d + x)^2 + x(d' - x)$. Also $(y - a)/d = (d + x)/y$. Eliminating x, we get $y^2 - 2a'y - d^2 = 0$, where $a' = a(2d + d')/2(d + d')$. Hence,

$$y = a' \pm \sqrt{a'^2 + d^2}.$$

The analysis suggests the following construction: With B as a center, BC as a radius, describe the arc cutting the horizontal diameter produced of the lower circle in D. Draw BD cutting the common tangent in E. Take $EF = a$. Draw FG perpendicular to AC. Then AG will equal a'. Make $AH = AG$. Draw BH and produce it to J, making $HJ = AH$. With B as a center and BJ as radius, describe an arc cutting the lower circle in P. Then BP is the required secant.

Also solved by P. J. DA CUNHA.

2741 [1919, 35]. Proposed by H. L. OLSON, New Hampshire College.

Prove or disprove the following statement: If the three sides and the area of a triangle are integers, at least one of the three altitudes is an integer.

I. SOLUTION BY FRANK IRWIN, University of California.

The statement is not true since the triangle whose sides are 5, 29, 30 has area 72, and altitudes 144/5, 144/29, and 24/5.

II. REMARKS BY J. L. RILEY, Stephenville, Texas.

On page 12 of Carmichael's *Diophantine Analysis* we find the following theorem:

A necessary and sufficient condition that rational numbers x, y, z shall represent the sides of a rational triangle is that they shall be proportional to numbers of the form $n(m^2 + h^2)$, $m(n^2 + h^2)$, $(m + n)(mn - h^2)$, where m, n, h are positive rational numbers and $mn > h^2$.

In deriving this result it is pointed out that if $x = n(m^2 + h^2)$, $y = m(n^2 + h^2)$, and $z = (m + n)(mn - h^2)$, the area is $hmn(m + n)(mn - h^2)$ and the altitude upon z is $2hmn$. If m, n, and h are integers ($mn > h^2$) we have a series of triangles for which the statement of Mr. Olson is true.

2742 [1919, 36]. Proposed by C. N. SCHMALL, New York City.

In Gregory's *Examples in the Differential and Integral Calculus*, 1841, Chap. VII, p. 124, ex. 22, I find the following celebrated problem: "To find a point within a triangle from which if lines be drawn to the angular points their sum may be the least possible." The author remarks that "the direct solution of this problem is long and complicated, etc." Required a simple, brief solution.

I. SOLUTION BY F. V. MORLEY, Johns Hopkins University.

Consider the three points, α, β, γ (complex variable) as on a unit or base circle. Then th sum of the distances from the point x is

$$\sum^3 \sqrt{(x - \alpha)\left(\bar{x} - \frac{1}{\alpha}\right)} \qquad (1)$$